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Procedia Engineering 144 (2016) 26 – 33

Procedia Engineering

www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Conditions of Visibility of Bridge Natural Frequency in Vehicle Vertical Acceleration

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Abstract

For structural health monitoring, selected bridges are generally instrumented and recorded data are utilized to obtain the natural frequency. However, this method is expensive and cannot cover all the bridges. Hence an alternative way for finding the bridge natural frequency is to utilize the vertical acceleration of moving vehicle. In the present paper spectrogram of vehicle vertical acceleration obtained theoretically has been analyzed to determine the conditions for which fundamental or higher mode bridge frequencies are visible. A flexible vehicle model moving along a simply supported bridge has been analyzed. Effect of vehicle / bridge mass ratio and surface roughness conditions on the visibility of bridge natural frequency have been investigated.

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: *Natural frequency, structural health monitoring, spectrogram, flexible-vehicle model*

1. Introduction

Natural frequency of the structures proves to be a reliable parameter in condition monitoring of structures as it reflects the reduction of stiffness due to damage when it drops down [1]. Bridge natural frequencies are usually determined by instrumenting a bridge with sensors and thereafter post processing the acquired sensor data. This method is a direct approach and costly. Moreover, all the bridges cannot be instrumented to acquire vibration data whereas vehicle fitted with an accelerometer can travel over different bridges. Identification of bridge natural frequency from vehicle acceleration data has been studied by several authors using acceleration spectra [2,3]. It may be noted that this indirect method of determining bridge fundamental frequency from moving vehicle acceleration is

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.003

an attractive option; however, due to certain conditions of bridge deck and lower vehicle mass, results may not be promising [4]. Nguyen and Tran [5] apply a Symlet wavelet transform to the displacement response of a moving vehicle to identify the existence and location of cracks in a bridge. One of the most recent attempts at extraction of bridge frequency from a passing vehicle is based on optimization. Li et al. [6] develop a new theoretical method based on the Generalized Pattern Search Algorithm (GPSA) which is a typical search method in optimization. In the present paper, a flexible vehicle-bridge interaction model has been developed and solved using analytical technique with the help of symbolic computational software MATHEMATICA. Spectrogram of the vehicle acceleration has been utilized to detect bridge fundamental frequency.

2. Bridge-Vehicle Coupled System Model

In most of the past research, vehicle body has been modeled as rigid body. The model has been improved considering bending of the vehicle body in the present study. The full vehicle body along the length has been represented as Euler- Bernoulli beam with bending flexibility. The length of the vehicle is lv. The analysis is limited to the linear suspension characteristics. The bridge-vehicle model has been shown in Fig. 1.

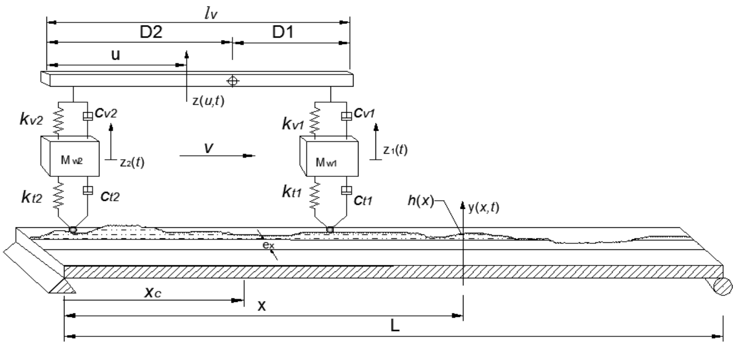


Fig. 1 Bridge subjected to Half Car vehicle Model

Vehicle centroid is located by the distance D_1 and D_2 measured from the trailing and leading edge of the vehicle body respectively. The governing differential equation of motion of the vehicle vertical deflection can be expressed as [7]

$$E_v I_v \frac{\partial^4 z(u,t)}{\partial u^4} + C_v \frac{\partial z(u,t)}{\partial t} + m_v \frac{\partial^2 z(u,t)}{\partial t^2} = f_v(u,t) \tag{1}$$

in which mv denotes the mass per unit length, , $E_v I_v$ is the flexural rigidity and C_v is viscous damping per unit length of the vehicle body, $z(u,t)$ represents vertical deflection of the vehicle body measured at location u from the reference point (taken at the left end of the vehicle) at time instant t. The vertical force imposed on the vehicle body is given by

$$f_{vF}(u,t) = \sum_{j=1}^{2} [k_{vj}\{z(u,t) - z_j(t)\} + c_{vj}\{\dot{z}(u,t) - \dot{z}_j(t)\}]\delta(u - u_j) \tag{2}$$

u_j represent the location of the attachment point of suspension from the reference point, $z_j(t)$ denote the vertical displacement of wheel, k_{vj} are the vehicle suspension stiffness, c_{vj} are the vehicle suspension damping. The subscript j =1, 2 represents the suspension location, for example j =1 denotes all quantities related to front suspension and j =2 for rear suspension. The equations of motion for two un-sprung masses are given by

$$
\begin{aligned}
& m_j \ddot{z}_j(t) + k_{tj}\{z_j(t) - y(x_j,t) - h(x_j)\} + k_{vj}\{z_j(t) - z(u_j,t)\} \\
& \quad + c_{tj}\{\dot{z}_j(t) - \dot{y}(x_j,t) - \dot{h}_i(x_j)\} + c_{vj}\{\dot{z}_j(t) - \dot{z}(u_j,t)\} = 0
\end{aligned}
\qquad (j=1,\,2) \tag{3}
$$

The governing differential equation of motion of the bridge in flexure and torsion can be expressed as

$$
E_b I_b \frac{\partial^4 y(x,t)}{\partial x^4} + C_b \frac{\partial y(x,t)}{\partial t} + m_b \frac{\partial^2 y(x,t)}{\partial t^2} = f_b(x,t) \tag{4}
$$

in which m_b is the mass per unit length, C_b is viscous damping per unit length whereas E_bI_b represents flexural rigidity of the bridge. The imposed vertical force $f_b(x,t)$ on the bridge due to vehicle interaction is given by

$$
\begin{aligned}
f_b(x,t) = & -\sum_{j=1}^{2}[k_{tj}\{z_j(t) - y(x,t) - h(x_j)\} + c_{tj}\{\dot{z}_j(t) - \dot{y}(x,t) - h(x)\}]\delta(x - x_j) \\
& -\sum_{j=1}^{2}\{m_{wj} + \frac{1}{2}m_v l_v\} g\,\delta(x - x_j)
\end{aligned}
\tag{5}
$$

The governing differential equation of torsional vibration of bridge can be written as

$$
G_b J_b \frac{\partial^2 \gamma(x,t)}{\partial x^4} - C_{bT} \frac{\partial \gamma(x,t)}{\partial t} - I_{mb} \frac{\partial^2 \gamma(x,t)}{\partial t^2} = f_T(x,t) \tag{6}
$$

in which I_{mb}, represents the polar moment of inertia per unit length of bridge cross section, G_bJ_b is the torsional rigidity and C_{bT} is the rotational damping per unit length of bridge respectively. The distributed torque due to eccentricity can be expressed as

$$
\begin{aligned}
f_T(x,t) = & -\sum_{j=1}^{2}[k_{tj}\{z_j(t) - y(x,t) - h(x\ \ )\} + c_{tj}\{\dot{z}_j(t) - \dot{y}(x,t) - h(x)\}]e_x\,\delta(x - x_j) \\
& -\sum_{j=1}^{2}\{m_{wj} + \frac{1}{2}m_v l_v\} g\, e_x \delta(x - x_j)
\end{aligned}
\tag{7}
$$

In which $f_b(x,t)$ is the imposed vertical force and $f_T(x,t)$ is imposed torque on the bridge. k_{tj} and c_{tj} represent tyre stiffness and tyre damping at front axle (j =1) and rear axle (j =2) locations respectively. In the present study the bridge deck surface roughness has been realized as homogeneous process in spatial domain [8]. In the present study, first the partial differential equations (PDE) are discretized into ordinary differential equations (ODE) in generalized time dependent coordinates using mode superposition techniques as

$$
[M]\{\ddot{r}(t)\} + [C(t)]\{\dot{r}(t)\} + [K(t)]\{r(t)\} = \{F(t)\} \tag{8}
$$

where $\{r(t)\}=\{\eta_{v1}(t), \eta_{v2}(t),..., \eta_{nv}(t), z_1(t), z_2(t), \eta_{b1}(t), \eta_{b2}(t),..., \eta_{nb}(t), \gamma_1(t), \gamma_2(t),...,\gamma_{nT}(t)\}^T$ is the response vector, $\{F(t)\}$ is the generalized force vector and, $[M]$, $[C(t)]$ and $[K(t)]$ are system mass, damping and stiffness matrix respectively. The number of coupled equations is $n= 2+n_v+n_b+n_T$ where n_v=number of vehicle bending modes including rigid bounce and pitch, n_b=bridge bending modes, n_T=bridge torsional modes. The set of ODE (8) are then recast into state space form and decoupled using time dependent complex eigen modes [9]. A closed form solution of response integral has been derived for each input sample of force time history using symbolic package MATHEMATICA.

3. Spectrogram Analysis of Vehicle Response

The complexity of the bridge vehicle interaction problem arises mainly due to the space-time variant nature of the equations of motion. When the vehicle is on the bridge, their responses are coupled together and they cannot be analyzed independently. The system matrices are different for every position of the vehicle on the bridge. Thus, the natural frequencies of the combined system also vary during the vehicle crossing. However, this coupled nature of the two subsystems has been utilized in the present study for identification of particular sub-system's natural frequencies from another subsystem response. Once acceleration of a vehicle body is obtained from known state, spectrogram of the signal has been obtained and analyzed to check the appearance of bridge fundamental frequency. A *spectrogram* is a visual representation of the spectrum of frequencies in a time signal [10]. Spectrograms are extensively used in speech processing. Seismology is another area where this technique is found useful. A common format of spectrogram is a graph with geometric axes, the horizontal axis represents time while vertical axis represents frequency. A third dimension indicating amplitude at a particular frequency at particular time is represented by intensity or colour of each point in the image. The spectrogram (Φ_X) of a signal X(t) in time and frequency axis can be mathematically expressed as

$$\Phi_X(t,\omega) = \left| X^*(t,\omega) \right|^2 \tag{9}$$

where X* is short time Fourier transform of the signal. This is defined as

$$X^*(\tau,\omega) = \int_{-\infty}^{+\infty} X(t)W(t-\tau)\exp(-j\omega t)dt \tag{10}$$

in which X(t) is the signal to be transformed. W(t) is the window function. In common practice, Hanning window or Gaussian window centered on zero is chosen.

4. Results and Discussions

4.1. Bridge Responses

In a Bridge-Vehicle interaction problem, bridge deflection and road surface roughness are the two sources of vehicle excitation. The excitation frequency from pavement unevenness depends linearly on the constant vehicle driving speed. As a result, the acceleration or retardation of the vehicle increases or decreases the frequency of road excitation. The data of a RC slab –Girder type Bridge of span (L), 20m; three longitudinal girders and three cross girders one at mid span and other two at supports are adopted for the study. The cross section of the bridge is shown in Fig 2. The lane width: 8.6 m, deck thickness: 200 mm, concrete characteristic strength 25 N/mm^2. The following physical parameters are assigned to the bridge model for conducting parametric study: Mass (m_b): 11.15×10^3 kg/m, flexural rigidity (E_bI_b):3.7×10^{10} N-m^2. Modal damping ratio: 0.04. The important physical parameters pertaining to vehicle are given as, length (l_v): 12 m, axle spacing (wheel base): 9 m, flexural rigidity (E_vI_v): 5.3×10^6 N-m^2, Mass per unit length (m_v): 1500 kg/m, front and rear wheel masses (m_{w1},m_{w2}): 800 kg each, suspension stiffness front and rear (k_{v1}, k_{v2}): 3.6×10^7 N/m, suspension damping front and rear (c_{v1}, c_{v2}): 7.2×10^4 N-sec/m, front and rear tyre stiffness (k_{t1}, k_{t2}): 0.9×10^7 N/m, front and rear tyre damping (c_{t1}, c_{t2}): 0.7×10^4 N-sec/m. Damping ratio of vehicle body was 0.02. The variation of bridge dynamic response for the change in vehicle speed is shown in Fig.3. The mean peak central deflection as well as excitation frequency is found to increase when vehicle speed increases from 40 km/h to 80 km/h.

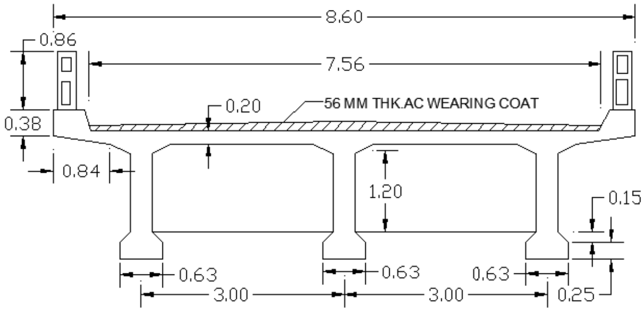
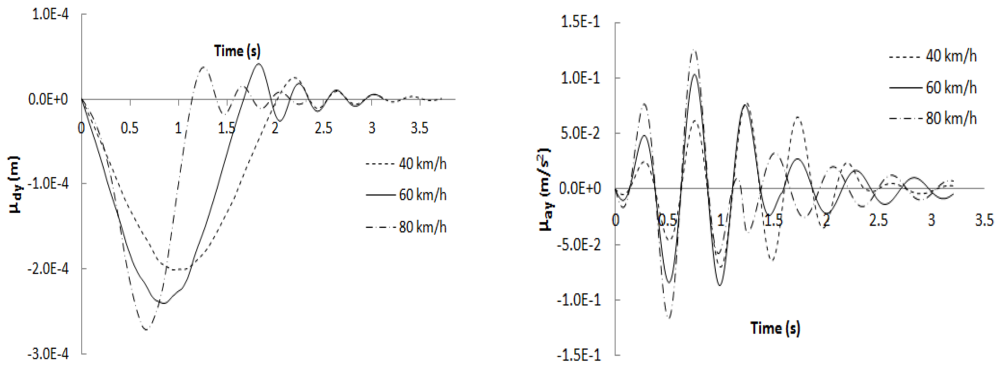


Fig. 2. Cross section of T-beam bridge

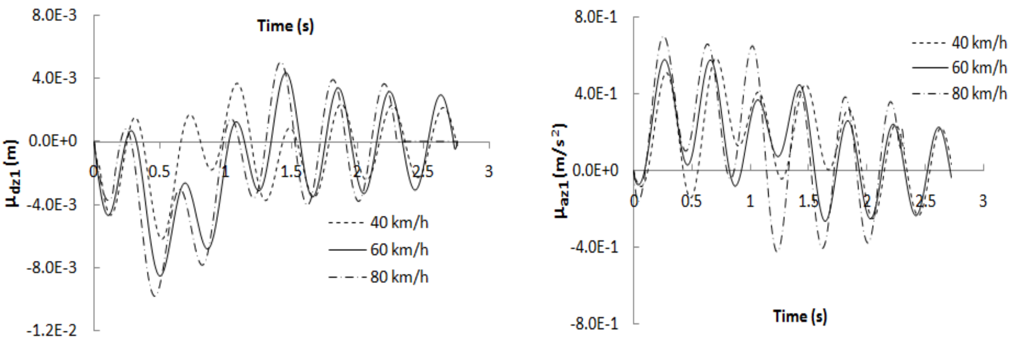


(a) Displacement (b) Acceleration

Fig. 3 Bridge responses at mid span

4.2. Vehicle Responses

Vehicle responses - displacement and acceleration at C.G are shown in Fig. 4. The peak responses are found to be increased with increase in vehicle speed. There is an increase of peak magnitude but frequency of oscillation is not very much affected.



(a) Displacement (b) Acceleration

Fig. 4 Vehicle body responses at CG

4.3. Identification of Bridge Natural Frequency

In the present analysis, spectrogram of vehicle acceleration response has been constructed using sampling frequency 200 Hz and 50% overlap between signal segments. Bridge natural frequencies for the first three modes of bending are 7.15 28.60 and 64.35 Hz respectively. The followings sections give the parametric studies conducted in search of bridge natural frequency from vehicle vertical acceleration.

4.3.1. Effect of Vehicle-Bridge mass ratio

After transforming eq.(8) in state space form, an eigen value analysis was carried out in complex domain considering different vehicle-bridge mass ratio. In complex eigenvalues, one can find out the damped natural frequencies considering real and imaginary part as shown by Inman [11]. Table-1 presents the effect of mass ratio on first three coupled frequencies of the bridge-vehicle. The orders in which coupled frequencies appear are dominated by vehicle frequencies (heave mode of vehicle, first bending mode of bridge and pitching of the vehicle). However, for health monitoring purpose of the bridge, search in the spectrogram would be carried out only for the bridge frequencies. The roughness of the bridge deck has been assumed to be of good category as per ISO classification corresponding to roughness coefficient ς_s=11×10–6 m^2/cycle /m [12]. System natural frequencies variation with time is given in the form of spectrogram. In the present study, spectrogram has been consrtucted from vehicle acceleration for constant vehicle speed 60 km/h. Fig. 5 and 6 show the spectograms of the vehicle c.g acceleration for speed 60 km/h and mass ratio 0.061 and 0.081 respectively.

For mass ratio 0.061, frequency is found to be localized closed to first mode of vehicle frequency 2.59 Hz. Second mode vehicle frequency is weekly visible. For higher mass ratio 0.081, frequency is localized at around 2.5 Hz and 7.2 Hz, which are vehicles first bouncing mode and first natural frequency of the bridge. Bridge second natural frequency is found to be even weakly visible for this mass ratio. This is apparent from Fig.6, as the highest strength of the signal indicated by dark red patch on the right hand side vertical axis coalesces with the frequency axis at the location of bridge first natural frequency.

Table 1. Coupled natural frequencies for various vehicle mass to bridge mass ratio (r)

Vehicle mass (×10^{3}kg)	**Bridge mass (×10^{3}kg)**	**`Mass ratio (*r*)**	**Coupled frequencies of the bridge-vehicle (Hz)**		
			1st mode	2nd mode	3rd mode
13.6	223.0	0.061	2.614	7.207	14.128
18.0	223.0	0.081	1.471	4.054	7.947
23.0	223.0	0.102	0.941	2.594	5.086

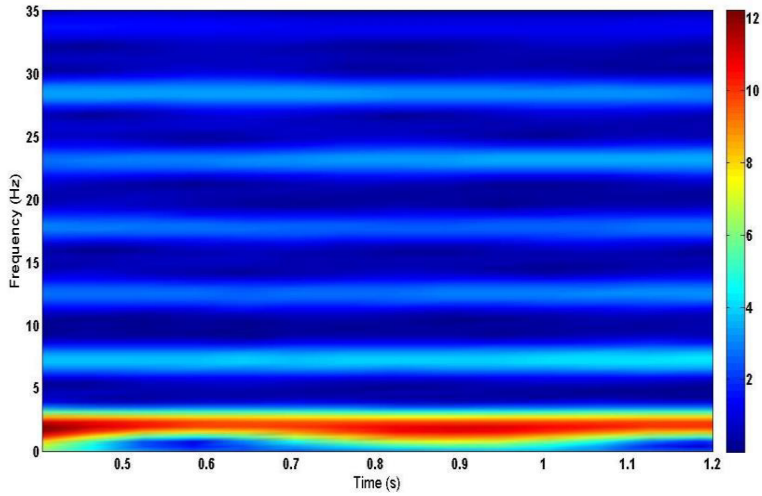
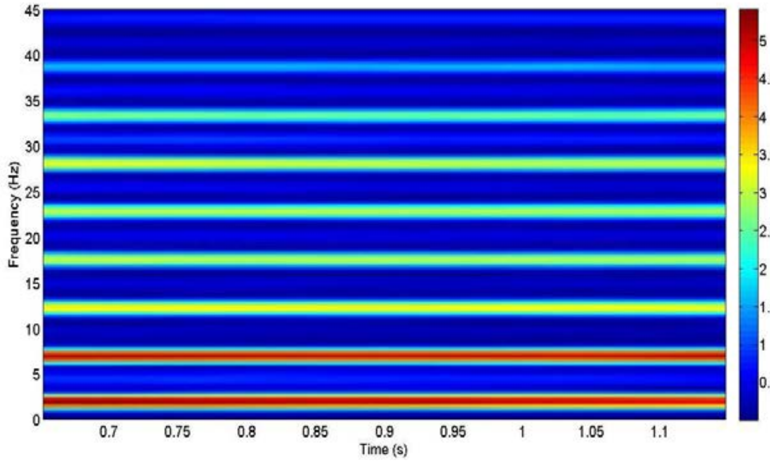


Fig.5 Spectrogram of vehicle acceleration response r = 0.061 in a good pavement condition



(b)

Fig. 6 Spectrogram of vehicle acceleration response r=0.081in a good pavement condition

4.4. 4.3.2 Effect of deck surface roughness

The previuos case study shows that bridge fundamental frequency is clearly visible for good condition of deck surface profile with vehicle mass ratio 0.081. Now, for the poor condition of deck surface, spectrogram of vehicle acceleration for the mass ratio 0.081 with vehicle speed 60 km/h are given in Fig. 7. Analysis of the result shows that visibility of bridge natural frequency from vehicle response spectogram is faded out by the poor category of road surface. In this figure, frequency is found to be localized at around 1.2 Hz and 7.5 Hz. The first value does not indicate either vehicle or bridge dominant mode. However, second value is more close to fundamental frequency of the bridge but is weakly visible as indicated by yellow coloured patch on frequency axis.

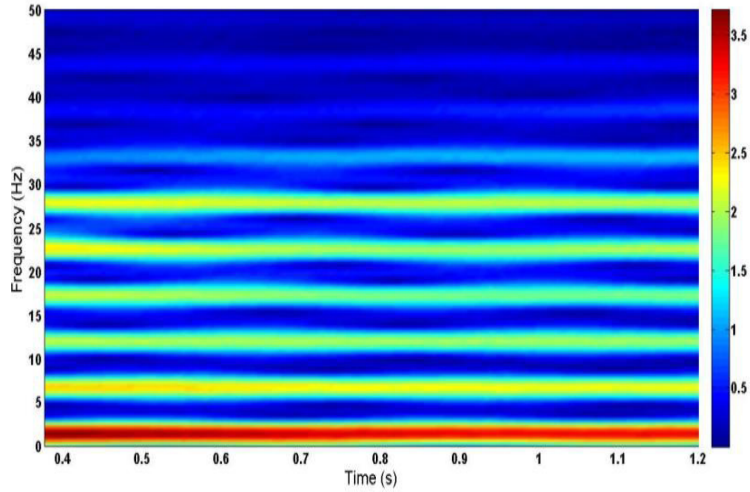


Fig. 7 Spectrogram of vehicle acceleration response for r = 0.081 in poor condition of bridge pavement

5. Conclusion

This paper presents a theoretical study on the visibility of bridge natural frequencies from vehicle response. Vehicle body flexibility has been introduced in the theoretical formulation. Spectrogram analysis has been performed on vehicle acceleration in search of bridge natural frequency. Importance of bridge-vehicle mass ratio has been observed from the study. For the present system parameters, mass ratio 0.081 gives optimal resolution to detect fundamental frequency of the bridge from vehicle acceleration. However, poor condition of bridge deck surface decreases the chance of appearing bridge fundamental frequency in spectrogram more clearly. This approach seems to be promising and cost effective if the influencing factors are favourable in field conditions. The instrumentation of vehicle is easier compared to that in a bridge and therfore several bridges can be investigated by the present method in a shorter time.

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